Polynomial chaos expansion for wave propagation

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Wave propagation in a random medium

The problem of wave propagation through a random medium arises naturally in many physical applications. Many theoretical works have been done on this subject but its numerical simulation requires a strategy to overcome the numerical cost limitation. In fact, a naive approach would be far too expensive and the construction of a metamodel often falter over the long term integration problem ([2]).

Moreover, in the case of wave propagation, the received signal results from the superposition of interfering wave packets, each one depending on the stochastic characteristics of the medium. To deal with the complexity of the resulting signal, we propose a method to build a metamodel based on a decomposition adapted to the medium: the normal modes of the propagating operator.

A stochastic basis adapted to the propagation

After a Fourier transform in time, the problem of wave propagation results in solving the Helmoltz equation:

$$\mathcal{H}(x,\xi)u = \Delta u + \frac{\omega^2}{c(x,\xi)^2}u = s(\omega) \tag{1}$$

where $s(\omega)$ is the spectrum of the source and $c(x,\xi)$ the wave celerity in the medium. The randomness of the medium gives a wave celerity which depends on random paramaters ξ.
Linear operator theory ensures that the eigenvectors $(\Psi_k)_{k\in K}$ of $\mathcal{H}$ form a basis of the space of squarely integrable functions. This basis gives a natural decomposition in wave packets for the solution u.
However, this basis is defined as the spectrum of a random operator $(\mathcal{H}(x,\xi))$ and depends on the stochastic parameters ξ. We propose to consider the Polynomial Chaos expansion (gPC) of this basis in order to be able to decompose the solution for every realisation of our medium with a low computational cost.

A modular metamodel

Once the gPC expansions of the normal modes $(\widehat{\lambda_k}(\omega,\xi))_{k\in K}$ and $(\widehat{\Psi_k}(x,\omega,\xi))_{k\in K}$ are computed, they can be used to generate signals for a given source at a distance R:

$$u(\omega,R,\xi) = \left[\frac{i}{4}\sum_{k\in K} H_0^{(1)}(\widehat{\lambda_k}(\omega,\xi)R)\widehat{\Psi_k}^2(0,\omega,\xi)\right]s(\omega) \tag{2}$$

Since the metamodel is build upstream, a stochastic source can be considered without supplementary cost. Sensitivity analysis can also be conducted using those expansions.

Moreover, this appraoch gives a natural framework for model reduction: the sum can focus on the most contributing modes ([1]). For instance, by taking only one mode we have a metamodel for one wave packet which can be usefull when studying a particular arrival in a received signal.

Towards a multi-level approach

The wide range of possible perturbations can alter the convergence of the gPC expansion. We propose to seperate two scales of the perturbation: the large scales structures which are treated as described above and small-scale structures treated with a perturbative approach.

This perturbative approach relies on the coupling matrix between the acoustic modes. This coupling matrix depends on the large scale structures but its gPC expansion can be deduced from the expansions $(\widehat{\lambda_k}, \widehat{\Psi_k})_{k \in K}$.

References

[1] Michael Bertin, Christophe Millet, and Daniel Bouche. A low-order reduced model for the long range propagation of infrasounds in the atmosphere. *The Journal of the Acoustical Society of America*, 136(1):37–52, 2014.

[2] Xiaoliang Wan and George Karniadakis. Long-term behavior of polynomial chaos in stochastic flow simulations. *Computer Methods in Applied Mechanics and Engineering*, 195:5582–5596, 08 2006.

Short biography – This PhD has started in January 2017 at ENS Paris-Saclay and is funded by CEA. CEA is working with the CTBTO (Comprehensive Nuclear Test Ban Treaty Organization) on the detection of explosions on the surface of the globe using infrasound monitoring stations. The aim of this work is to take into account the impact of the atmospheric unertainties on infrasound propagation.