Extended Principal Component Analysis algorithm for adaptive model reduction in inverse problems

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Ph.D. expected duration: Sep. 2019 - Jul. 2023

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Abstract:

The problem of identifying model parameters given observed data appears in many areas of contemporary research. Such problems are called inverse problems and in most cases are ill-posed. The main approach for fast and efficient solution of this type of problems is to solve the corresponding optimization problem using variety of optimization techniques. Objective function in such optimization problems is typically written in form of squared misfit between observed data and model simulation results:

$$\min_{x} \left[F(x) = \sum_{i=1}^{N} \left\{ \frac{f(x_i) - d_i^{obs}}{\sigma_i} \right\}^2 \right] \tag{1}$$

where d_i^{obs} - observed data, x - model, $f(x_i)$ - function of model, σ_i - standard deviation.

In practical problems $f(x)$ is determined by physical processes modelling and has form of system of PDE's which is commonly packed into complex simulation systems or software products. Calculation of $f(x)$ is referred as solving forward problem. In practice, one run of the forward problem could take from few ms to few days of computation cost.

One of the main approaches to accelerate the convergence of an optimization algorithm for problem (1) is to parametrize model x or, in other words, to reduce model size. Typically this reduction is achieved by decomposition of the initial model using an orthogonal basis and further optimization in reduced space of decomposition parameters.

The model reduction problem has been widely studied in the last 30 years. In general, most of parametrization techniques can be classified into two groups [2]. First represents model decomposition as linear combination of fixed basis functions (e.g. Discrete Cosine Transform, Discrete Wavelet Transform), while the second group allows determination of the basis functions by given dataset of prior information (e.g. PCA-based techniques [5] or, which is the same in this context, Karhunen-Loève expansion and its variations [3]). In majority of problems, where a model has complex structure (e.g. has to be physically consistent) the family of PCA-based techniques is used as methods which allow preserving physical consistency by incorporating two-point or multi-point correlations between elements of prior dataset [3]. Model reduction based on Karhunen-Loève expansion is also a regularization approach for inverse problems based on stochastic modelling. However, these methods still have disadvantages caused by sole using of prior information.

Key idea of this work is to reconstruct the PCA basis by introducing the information of objective function sensitivity into basis composition process.

Classic PCA basis provides best model decomposition in terms of minimizing total mean squared error of the approximation [1]:

$$\min_{\varphi_k} \mathbf{E}\Big[\int_r \big(\delta x(r)\big)^2 dr\Big] \tag{2}$$

$$\delta x(r) = x_\omega(r) - \tilde{x} = \sum_{k=1}^{\infty} A_k(\omega)\varphi_k(r) - \sum_{k=1}^{N} A_k(\omega)\varphi_k(r) = \sum_{k=N+1}^{\infty} A_k(\omega)\varphi_k(r) \tag{3}$$

where $\delta x(r)$ is the approximation error between model $x_\omega(r)$ and its projection $\tilde{x}$ onto hyperplane formed by basis function $\varphi_k(r)$. $A_k(\omega)$ are corresponding coefficients of the model decomposition.

Main proposition of this work is to include objective function sensitivity information into (2) as:

$$\min_{\varphi_k} \mathbf{E}\Big[\int_r \big(\delta x(r)\big)^2 dr + \gamma \delta F\Big] \tag{4}$$

where δF is difference between objective function of initial model and approximated model:

$$\delta F(x) = F(x) - F(\tilde{x}) = F(\tilde{x} + \delta x) - F(\tilde{x}) \tag{5}$$

As a result, the efficient numerical algorithm for calculation of update for the PCA basis was derived. The technique is easy to implement in combination with any parametrization algorithm from PCA family. It also doesn't significantly increase computational cost of the whole workflow of solving inverse problem (1). The key benefit of this technique is improved performance of classic PCA-based techniques in presence of complex model structure and improper prior info dataset.

The result was achieved by applying quantum mechanics Perturbation Theory and adjoint technique for gradient calculation [4]. In presentation, key examples of applications in History Matching problem and comparative analysis with other popular parametrization techniques (DCT, DWT, PCA family) will be provided.

References

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Short biography – A. Mukhin got his B.S degree from the Moscow Institute of Physics and Technology (MIPT) and is currently enrolled in M.S program in MIPT co-funded by MIPT Center for Engineering and Technology. His topic focuses on the solving complex inverse problems such as history matching.