Robust Uncertainty Quantification of a Risk Measurement from a Computer Code

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Abstract:

Uncertainty quantification methods address problems related to with real world variability. Generally, an engineering system is represented by a numerical function $Y = G(X)$, whose inputs $X \in \mathbb{R}^p$ are uncertain and modeled by random variables. The variable of interest is the scalar output of the computer code, but the statistician rather work with some quantities of interest, for example, a quantile, a probability of failure, or any measures of risk. Uncertainty quantification aims to characterize how the variability of a system and its model affect the quantity of interest [1].

We propose to gain robustness on the quantification of this measure of risk. Usually input values are simulated from an associated joint probability distribution. This distribution is often chosen in a parametric family, and its parameters are estimated using a sample and/or the opinion of an expert. However the difference between the probabilistic model and the reality induces uncertainty. The uncertainty on the input distributions is propagated to the quantity of interest, as a consequence, different choices of input distributions will lead to different values of the risk measures.

To consider this uncertainty, we propose to evaluate the maximum risk measure over a class of distributions. Different classes are suggested in the literature mainly discussed in the work of Berger and Hartigan in the context of Robust Bayesian Analysis (see [10]). They consider for example the generalized moment set [4] or the ε-contamination set [11]. The generalized moment set has some really nice properties studied by Winkler [13] based on the well known Choquet theory [3]. An extension of Winkler's work has been more recently published by Owhadi and al. [9] under the name of Optimal Uncertainty Quantification (OUQ). In our work, we will focus on classes of measures specified by classical moment constraints. This is a particular case of the framework introduced by [9] justified by our industrial context, mainly related to nuclear safety issues [12]. Indeed, in practice the estimation of the input distributions, built with the help of the expert, often relies only on the knowledge of the mean or the variance of the input variables.

The solution of our optimization problem is numerically computed thanks to the OUQ reduction theorem ([9], [13]). This theorem states that the maximum of the risk measure is located on the extreme points of the distribution set. In the context of the moment class, it corresponds to a product of discrete finite measures. To be more specific it holds that when N pieces of information are available on the moments of a measure μ, it is enough to pretend that the measure is supported on at most $N + 1$ points.

One of the main issues is the computational complexity of the optimization of the risk measure over the given class of distribution. In the moment context, Semi-Definite-Programming [6] has been already already explored by Betrò [2] and Lasserre [7], but the deterministic solver rapidly reaches its limitation as the dimension of the problem increases. One can also find in the literature

a Python toolbox developed by McKerns and al. [8] called Mystic framework that fully integrates the OUQ framework. However, it was built as a generic tool for generalized moment problems and the enforcement of the moment constraints is not optimal. By restricting the work to classical moment sets, we propose an original and practical approach based on the theory of canonical moments [5]. Canonical moments of a measure can be seen as the relative position of its moment sequence in the moment space. It is inherent to the measure and therefore present many interesting properties. Our algorithm shows very good performances and great adaptability to any constraints order. The dimension is subject to the curse of dimension but can be perform up to dimension 10 for a reasonable cost.

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Short biography – The industrial needs of statistical robustness are required by the IRSN in the context of nuclear safety. This PhD originates from the close colaboration of EDF R&D and the University of Toulouse III - Paul Sabatier. Key words: *robustness, uncertainty quantification, optimization, canonical moments.*