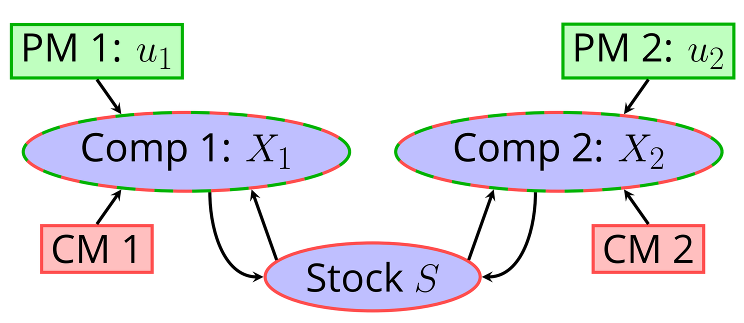
Maintenance optimization of large systems with a decomposition method

T. Bittar[1,2], P.Carpentier[1], J-Ph. Chancelier[1], J. Lonchampt[2]

[1]CERMICS, École des Ponts ParisTech, [2]EDF R&D, PRISME Department

Industrial problem: maintenance optimization

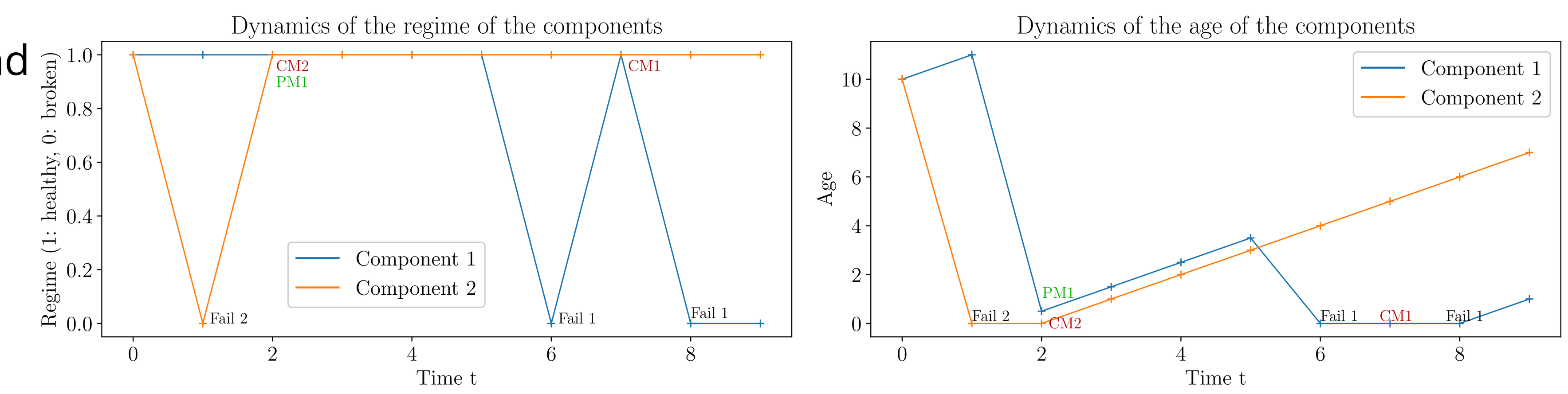
- Physical system with large number of components (turbines, alternators) and a common stock of spares



PM: Preventive maintenance (planned)

CM: Corrective maintenance (due to random failures)

Sketch of the dynamics of a system with 2 components and a stock



Example of the dynamics of the regime and the age of 2 components : failures at $t = 6, 8$ for comp 1 and at $t = 1$ for comp 2. CMs at $t = 7$ for comp 1 and $t = 2$ for comp 2. PM at $t = 2$ for comp 1

- Stochastic dynamics because of the random failures of the components
- Stochastic cost because of the random CMs and forced outages cost

Main goal

Find the deterministic maintenance policy that minimizes the expectation of the cost i.e. find the best PM dates for each component

1. Formalization of the problem

- System of n components on time horizon T
  - High dimension of the decision set $\mathcal{U}$
- Several couplings:
  - In the dynamics of the system (stock)
  - In the forced outage cost j^{FO}

The maintenance optimization problem

$$\min_{(X,S,u)\in\mathcal{X}\times\mathcal{S}\times\mathcal{U}} \mathbb{E}\Big[\underbrace{\sum_{t=0}^{T}\sum_{i=1}^{n} j_{i,t}(X_{i,t},u_{i,t})}_{Additive} + \underbrace{j_T^{FO}(X_t)}_{Coupling}\Big] \quad (1)$$

$$\text{s.t. } \underbrace{\Theta(X,S,u)}_{Coupling} = 0$$

- $j_{i,t}$: Maintenance cost (additive)
- j_t^{FO}: Forced outage cost (coupling)
- $X = (X_{i,t})_{i=1,\dots,n}^{t=0,\dots,T}$: State of the components
- $S = (S_t)_{t=0,\dots,T}$: State of the stock
- $u = (u_{i,t})_{i=1,\dots,n}^{t=0,\dots,T}$: Open loop control on the system (maintenance or not at t for component i)
- Θ: Vector gathering the system dynamics

2. Decomposition for large systems

Idea: Iteratively find the best policy for each component separately, then coordinate the components

2.1 Choice of an auxiliary problem [1]

- Additive auxiliary cost function $K : \mathcal{X}\times\mathcal{U}\to\mathbb{R}$
- Block-diag. auxiliary dynamics $\Phi : \mathcal{X}\times\mathcal{S}\times\mathcal{U}\to\mathbb{R}^p$
- Auxiliary problem with parameter $\bar{Z}\in\mathcal{X}\times\mathcal{S}\times\mathcal{U}$:

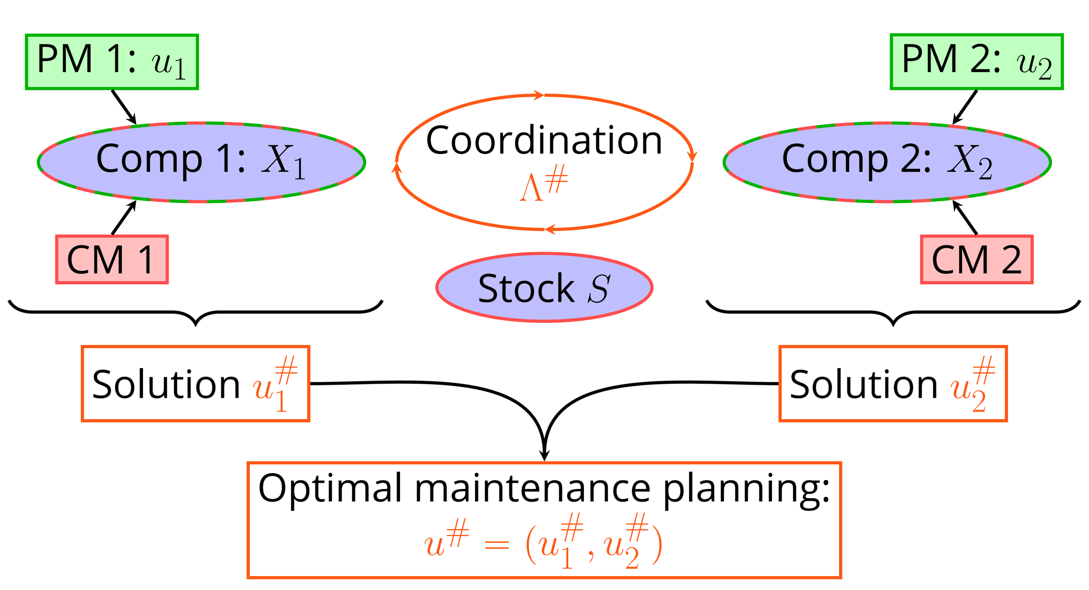
$$\min_{\substack{Z\in\mathcal{X}\times\mathcal{S}\times\mathcal{U}\\ Z=(X,S,u)}} \mathbb{E}\Big[\sum_{i=1}^{n}\sum_{t=0}^{T}\Big(j_{i,t}(X_{i,t},u_{i,t}) + K_{\bar{Z},i,t}(X_{i,t},u_{i,t})\Big) + \underbrace{\langle\bar{\Lambda},\ (\Theta'(\bar{Z}) - \Phi'_{\bar{Z}}(\bar{Z}))\cdot Z\rangle}_{Coordination}\Big]$$

$$\text{s.t. } \Phi_{\bar{Z}}(Z) = 0 \quad (2)$$

Fundamental lemma

If $Z^{\#}$ is solution of (2), and $Z^{\#} = \bar{Z}$ then it is a solution of (1).

Sketch of the principle of the decomposition method



Interest of the method

The auxiliary problem can be decomposed into n independent subproblems of smaller dimensions that can be solved in parallel :

dimension $nT \longrightarrow n\times$ dimension T

Algorithm 1 Fixed point algorithm

1: Start with $\bar{Z} = Z^0$, $\bar{\Lambda} = \Lambda^0$ and set $l = 0$

2: At iteration $l+1$, solve

$$\min_{\substack{Z\in\mathcal{X}\times\mathcal{S}\times\mathcal{U}\\ Z=(X,S,u)}} \mathbb{E}\Big[\sum_{i=1}^{n}\sum_{t=0}^{T}\Big(j_{i,t}(X_{i,t},u_{i,t}) + K_{Z^l,i,t}(X_{i,t},u_{i,t})\Big) + \langle\bar{\Lambda},\ (\Theta'(Z^l) - \Phi'_{Z^l}(Z^l))\cdot Z\rangle\Big]$$

$$\text{s.t. } \Phi_{Z^l}(Z) = 0$$

with an instance of MADS for each of the n independent subproblems. Solution Z^{l+1}

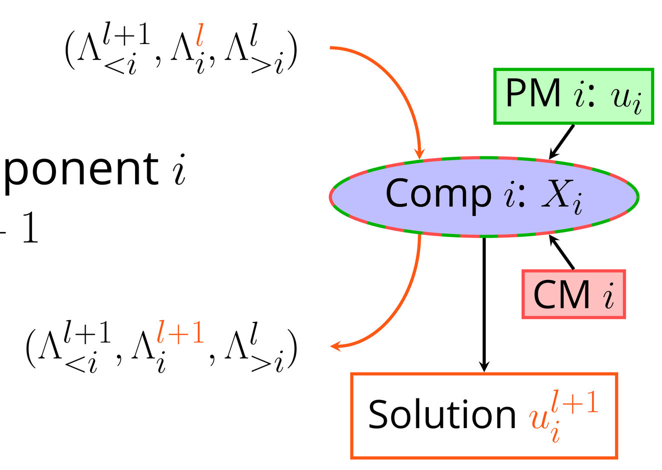
3: Compute an optimal multiplier Λ^{l+1} for the constraint $\Phi_{Z^l}(Z) = 0$ using the adjoint state

4: If $\|Z^{l+1} - Z^l\| + \|\Lambda^{l+1} - \Lambda^l\|$ is "sufficiently small" stop, else $l \leftarrow l+1$ and go back to 2

- Study of the convergence of the algorithm in [1].

2.2 Subproblem resolution: MADS

Subproblem on component i at iteration $l+1$



Mathematical formulation:

$$\min_{(X_i,u_i)\in\mathcal{X}_i\times\mathcal{U}_i} \mathbb{E}\Big[\sum_{t=0}^{T}\Big(j_{i,t}(X_{i,t},u_{i,t}) + K_{Z^l,i,t}(X_{i,t},u_{i,t})\Big) + \langle\bar{\Lambda},\ \nabla_{X_i}(\Theta(Z^l) - \Phi_{Z^l}(Z^l))\cdot X_i\rangle\Big]$$

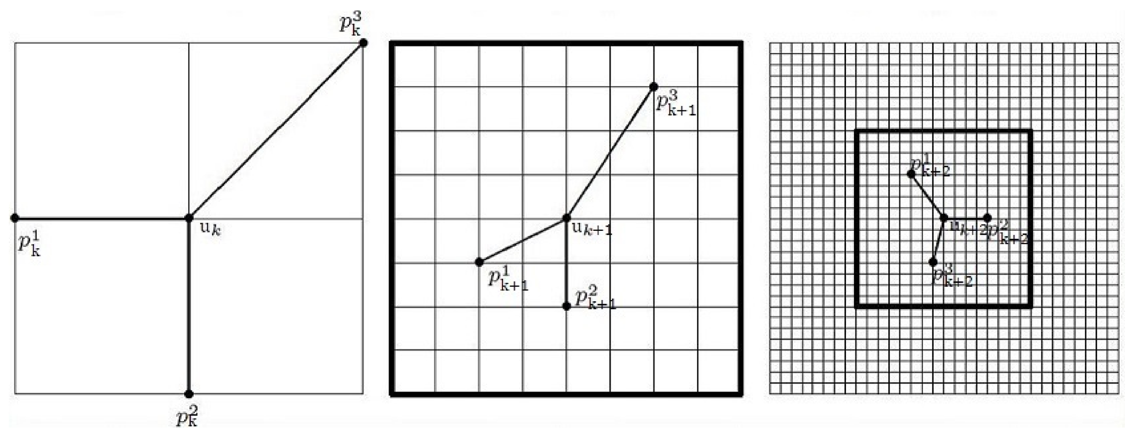
$$\text{s.t. } \Phi_{Z^l,i}(X_i,u_i) = 0$$

Blackbox framework

- Evaluation of the objective function is costly
- Assume no information on its gradients

Mesh Adapative Direct Search: At iteration k (of the subproblem resolution), define a mesh M_k, then:

- Global search: Flexible step: use of heuristics to choose evaluation points on M_k
- Local poll: Evaluation points chosen in a neighborhood $P_k \subset M_k$ of the current best solution
- Mesh update: If better solution found during search or poll then increase mesh parameter, else decrease it



MADS iterations: Points of M_k are all the points of the grid, points of P_k are inside the black square, taken from [2]

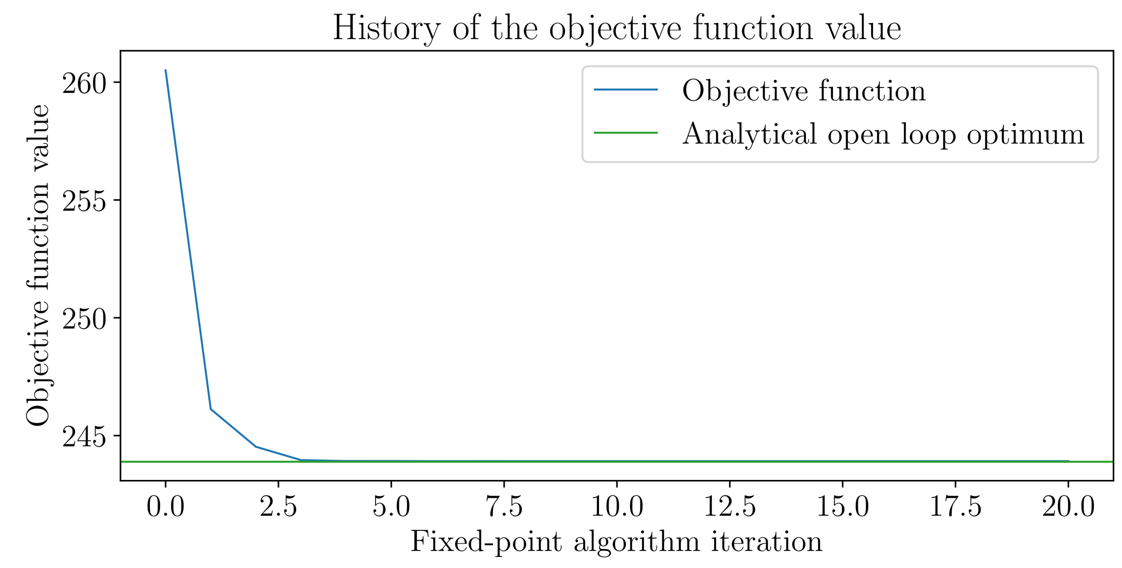
3. Numerical tests

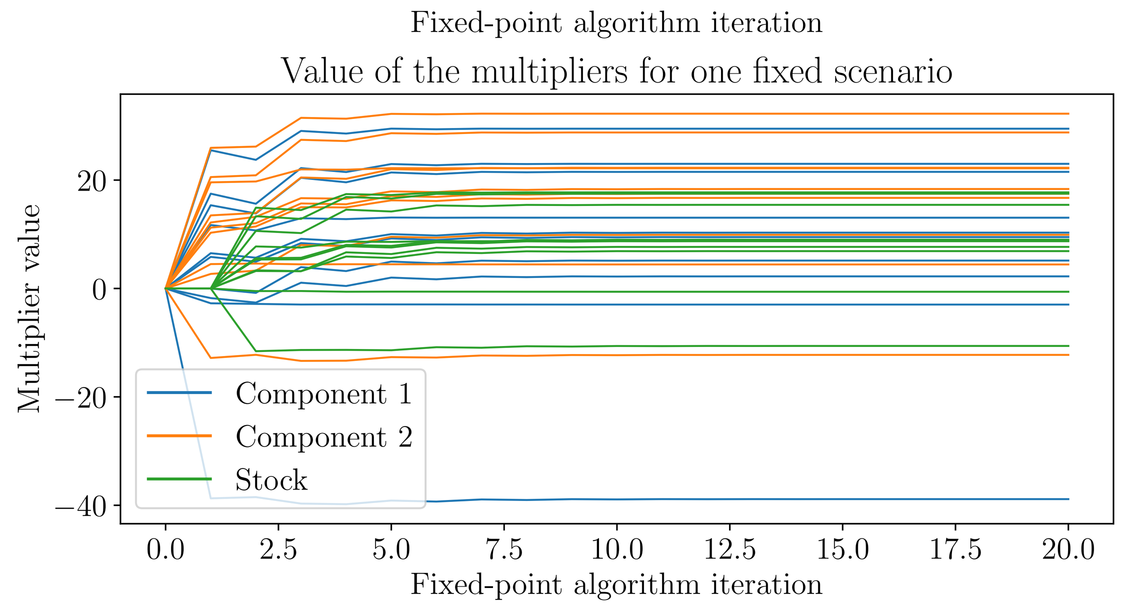
3.1 Application to a toy problem

- Linear system with quadratic costs, state space of dimension 3

$\Rightarrow$ Analytical solution available

- Same structure of couplings as in the maintenance problem





3.2 Difficulties arising in problem (2)

- Presence of integer variables (number of spare parts, physical state of the components)
- Complex non-smooth dynamics

$\Rightarrow$ Fixed point algorithm on a relaxed system: smoothing of dynamics and cost (in progress)

References

[1] G. Cohen. Auxiliary problem principle and decomposition of optimization problems. *Journal of Optimization Theory and Applications*, 32(3):277–305, November 1980.

[2] Charles Audet and J. E. Dennis, Jr. Mesh Adaptive Direct Search Algorithms for Constrained Optimization. *SIAM Journal on Optimization*, 17(1):188–217, 2006.