### Industrial problem: maintenance optimization

- Physical system with large number of components (turbines, alternators) and a common stock of spares
- Stochastic dynamics because of the random failures of the components
- Stochastic cost because of the random CMs and forced outages cost

1. **Formalization of the problem**

- System of \( n \) components on time horizon \( T \)
  - High dimension of the decision set \( U \)
- Several couplings:
  - In the dynamics of the system (stock)
  - In the forced outage cost \( f^{(2)} \)

#### The maintenance optimization problem

\[
\min_{X(S,u) \in U} \mathbb{E} \left[ \sum_{t=1}^{T} \left( \sum_{i=1}^{n} \left( j_{i,t}(X_i, u_{i,t}) + \frac{j^{(2)}_{i,t}}{\Lambda_i} \right) + l^{(1)}(X_t) \right) \right]
\]

s.t. \( \Theta \in X(S,u) = 0 \)

#### Choice of an auxiliary problem [1]

- **Additive auxiliary cost function** \( K : X \times U \rightarrow \mathbb{R} \)
- **Block-diag. auxiliary dynamics** \( \Phi : X \times S \times U \rightarrow \mathbb{R}^2 \)
- **Auxiliary problem with parameter** \( Z : X \times S \times U \)

\[
\min_{Z \in X(S,u)} \mathbb{E} \left[ \sum_{t=1}^{T} \left( \sum_{i=1}^{n} \left( j_{i,t}(X_i, u_{i,t}) + K_{i,t}(X_i, u_{i,t}) \right) + l^{(1)}(X_t) \right) \right]
\]

s.t. \( \Phi(Z) = 0 \)

2. **Decomposition for large systems**

**Idea:** iteratively find the best policy for each component separately, then coordinate the components

#### 2.1 Choice of an auxiliary problem [1]

- Additive auxiliary cost function \( K : X \times U \rightarrow \mathbb{R} \)
- Block-diag. auxiliary dynamics \( \Phi : X \times S \times U \rightarrow \mathbb{R}^2 \)
- Auxiliary problem with parameter \( Z : X \times S \times U \)

\[
\min_{Z \in X(S,u)} \mathbb{E} \left[ \sum_{t=1}^{T} \left( \sum_{i=1}^{n} \left( j_{i,t}(X_i, u_{i,t}) + K_{i,t}(X_i, u_{i,t}) \right) + l^{(1)}(X_t) \right) \right]
\]

s.t. \( \Phi(Z) = 0 \)

#### 2.2 Subproblem resolution: MADS

**Subproblem on component \( i \) at iteration \( l + 1 \)**

\[
\left( \lambda_{i}^{(l+1)}, \lambda_{i}^{(l+1)} \right) = \min_{Z \in X(S,u)} \mathbb{E} \left[ \sum_{t=1}^{T} \left( \sum_{i=1}^{n} \left( j_{i,t}(X_i, u_{i,t}) + K_{i,t}(X_i, u_{i,t}) \right) + l^{(1)}(X_t) \right) \right]
\]

s.t. \( \Phi(Z) = 0 \)

**Mathematical formulation:**

\[
\min_{Z \in X(S,u)} \mathbb{E} \left[ \sum_{t=1}^{T} \left( \sum_{i=1}^{n} \left( j_{i,t}(X_i, u_{i,t}) + K_{i,t}(X_i, u_{i,t}) \right) + l^{(1)}(X_t) \right) \right]
\]

s.t. \( \Phi(Z) = 0 \)

**Blackbox framework**

- Evaluation of the objective function is costly
- Assume no information on its gradients

**Mesh Adaptive Direct Search:** At iteration \( k \) (of the subproblem resolution), define a mesh \( M_k \), then:
  - **Global search:** Flexible step: use of heuristics to choose evaluation points on \( M_k \)
  - **Local poll:** Evaluation points chosen in a neighborhood \( M_k \subset M_{k+1} \) of the current best solution
  - **Mesh update:** If better solution found during search or poll then increase mesh parameter, else decrease it

#### 3. Numerical tests

**3.1 Application to a toy problem**

- **Linear system with quadratic costs, state space of dimension 3**
- **Analytical solution available**
- **Same structure of couplings as in the maintenance problem**

**3.2 Difficulties arising in problem (2)**

- Presence of integer variables (number of spare parts, physical state of the components)
- Complex non-smooth dynamics

**References**