Chance constraint optimization of a complex system

Application to the design of a floating offshore wind turbine

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1-Mathematical problem

Find a reliable optimum of the following problem :

$$\min_{x\in\Omega} \quad f(x)$$

$$\text{s.t} \quad \mathbb{P}_\xi(g_1(x,\xi)>0) < 10^{-4} \quad (1)$$

$$\mathbb{P}_{\xi,z}(\min_{t\in[0,T]} g_2(t,x,\xi;z)<0) < 10^{-4} \quad (2)$$

$$\mathbb{P}_{\xi,z}(\max_{t\in[0,T]} g_3(t,x,\xi;z)>0) < 10^{-4} \quad (3)$$

Uncertainties :
- ξ is a **random vector**
- z is a **random process piecewise stationary**

Objective function :
- f is a **smooth** and **explicit** function

Constraints :
- g_1 has strong non-linearities and is **expensive** to evaluate
- g_2 and g_3 are outputs of **linear filters**

Difficulties :

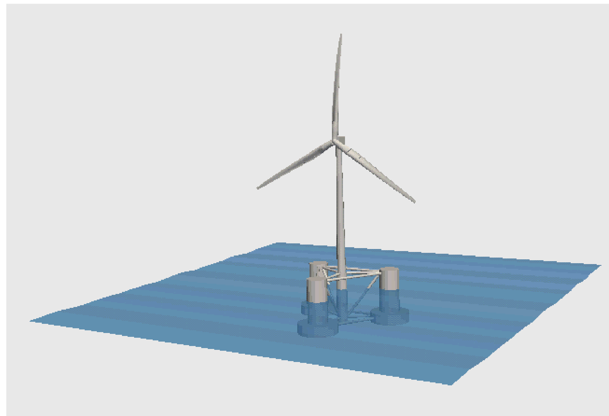
★ computation of **rare event** probabilities

★ **costly** performance function g_1

2-Industrial case : Floating Offshore Wind Turbine

Goal :

- minimize the **cost** of the anchoring system
- avoid **fatigue** rupture
- find a configuration of the mooring lines ensuring the station keeping of the wind turbine



NREL 5 MW - OC4-DeepCwind Semi-Submersible

The design variables are the **position of connection** of the lines to the floater, the **radius at the seafloor** and the **lineic mass** of a mooring line.

z models the uncertainties on the **sea elevation** and ξ the uncertainties of the **model**.

For the Fatigue Limit State, the constraints (computed with the software **Deeplines™**) are :

(1) the **fatigue** constraint : accumulated damage < resistance threshold ;

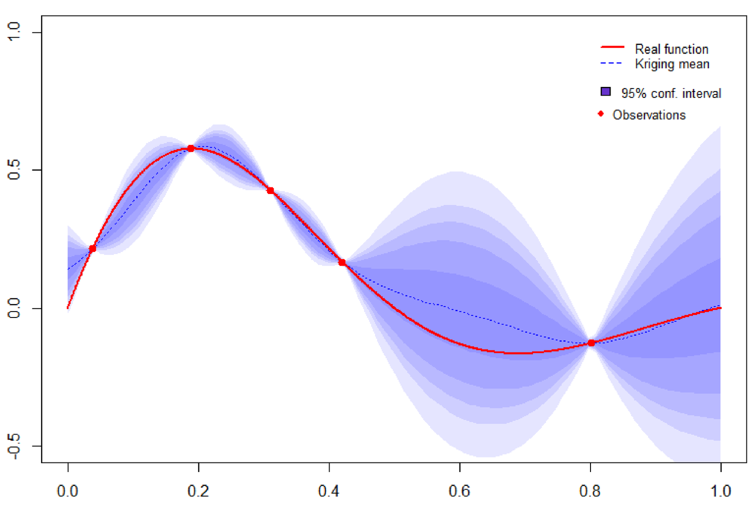
(2) the **tension** on the lines must stay positive ;

(3) the **pitch** must stay below 6 ° .

3-Fatigue constraint (1) : Meta-model

Several meta-models possible :

- **Kriging**
- **Polynomial chaos**
- Support vector machines
- Neural Networks



Different approaches to consider :

- meta-model ϕ of $x \to \log(\mathbb{P}_\xi(g_1(x,\xi)>0)$
- meta-model ψ of $(x,\xi) \to g_1(x,\xi)$

Active learning[1] ⇒ choice of refinement criteria

4-Constraints (2) and (3) : Extreme Value Theory

Suppose that ζ is a standardized stationary Gaussian process. Under some assumptions over the autocovariance function of ζ, we have the following result [2] :

$$\mathbb{P}\left(a_T\left(\max_{t\in[0,T]}\zeta(t)-b_T\right)\leq\alpha\right)\to\exp(-e^{-\alpha}) \quad \text{as} \quad T\to\infty$$

where a_T depends on T and b_T depends on T and on the second spectral moment of ζ.

At ξ fixed, g_2 and g_3 inherit the Gaussian property of z. The result above can then be applied to them.

5-Final formulation

At this stage, the constraints are reformulated as follows :

$$\min_{x\in\Omega} \quad f(x)$$

$$\text{s.t} \quad \phi(x) \text{ or } \log(\mathbb{P}_\xi(\psi(x,\xi)>0)) < \log(10^{-4})$$

$$\mathbb{E}_\xi\left[1-\exp\left(-e^{a_T\left(b_T(x,\xi)-\frac{\xi_5}{\sigma_1(x,\xi)}\right)}\right)\right] < 10^{-4}$$

$$\mathbb{E}_\xi\left[1-\exp\left(-e^{c_T\left(d_T(x,\xi)-\frac{\xi_6}{\sigma_2(x,\xi)}\right)}\right)\right] < 10^{-4}$$

The uncertainties on ξ can be dealt with Improved Monte Carlo methods (**Importance Sampling**, **Subset Simulation**...).

6-Optimization strategy

Different approaches :

- approaches based on FORM/SORM : double loop optimization(RIA, PMA) or single loop optimization (SORA, SAP [3])
- meta-model with adaptive design of experiments + optimization [4, 5, 6]
- Lagrangian formulation : $\min_{x,\lambda} f(x) + \lambda(\mathbb{E}_\xi[constraints(x,\xi)])$

7-Next step : add the Extreme constraint

Extreme constraint : $\mathbb{P}_{\xi,z}(\max_{t\in[0,T]} g_4(t,x,\xi;z)>0) < 10^{-4}$

- g_4 is nonlinear and costly to evaluate ⇒ Extreme Value Theory unusable
- With truncated Karhunen-Loève expansion :

$$\mathbb{P}_{\xi,z_1,\dots,z_m}(\max_{t\in[0,T]} g_4^{KL}(t,x,\xi;z_1,\dots,z_m)>0) < 10^{-4}$$

★ **High dimension**

References

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