Chance constraint optimization of a complex system

Application to the design of a floating offshore wind turbine

1-Mathematical problem

Find a reliable optimum of the following problem:

\[
\begin{align*}
\min_{x \in \Omega(t)} & \quad f(x) \\
\text{s.t.} & \quad P_\xi(g_1(t, x, \xi) > 0) < 10^{-4} \quad (1) \\
& \quad P_\xi(\max_{t \in [0,T]} g_2(t, x, \xi; z) < 0) < 10^{-4} \quad (2) \\
& \quad P_\xi(\min_{t \in [0,T]} g_3(t, x, \xi; z) > 0) < 10^{-4} \quad (3)
\end{align*}
\]

Uncertainties:
- \( \xi \) is a random vector
- \( z \) is a random process piecewise stationary

Objective function:
- \( f \) is a smooth and explicit function

Constraints:
- \( g_1 \) has strong non-linearities and is expensive to evaluate
- \( g_2 \) and \( g_3 \) are outputs of linear filters

Difficulties:
- computation of rare event probabilities
- costly performance function \( g_1 \)

2-Industrial case: Floating Offshore Wind Turbine

Goal:
- minimize the cost of the anchoring system
- avoid fatigue rupture
- find a configuration of the mooring lines ensuring the station keeping of the wind turbine

The design variables are the position of connection of the lines to the floater, the radius at the seafloor and the linear mass of a mooring line.

\( z \) models the uncertainties on the sea elevation and \( \xi \) the uncertainties of the model.

For the Fatigue Limit State, the constraints (computed with the software DEEPLINES\textsuperscript{TM}) are:

1. the fatigue constraint: accumulated damage < resistance threshold;
2. the tension on the lines must stay positive;
3. the pitch must stay below 6\(^\circ\).

3-Fatigue constraint (1): Meta-model

Several meta-models possible:
- Kriging
- Polynomial chaos
- Support vector machines
- Neural Networks

Different approaches to consider:
- meta-model \( \phi \) of \( x \to \log(P_\xi(g_1(t, x, \xi) > 0)) \)
- meta-model \( \psi \) of \( (x, \xi) \to g_1(x, \xi) \)

Active learning\cite{1} \( \Rightarrow \) choice of refinement criteria

4-Constraints (2) and (3): Extreme Value Theory

Suppose that \( \xi \) is a standardized stationary Gaussian process. Under some assumptions over the autocovariance function of \( \xi \), we have the following result \cite{2}:

\[
P \left( a_T \max_{t \in [0,T]} \zeta(t) - b_T \right) < \alpha \to \exp(-e^{-\alpha}) \text{ as } T \to \infty
\]

where \( a_T \) depends on \( T \) and \( b_T \) depends on \( T \) and on the second spectral moment of \( \zeta \).

At \( \xi \) fixed, \( g_2 \) and \( g_3 \) inherit the Gaussian property of \( z \). The result above can then be applied to them.

5-Final formulation

At this stage, the constraints are reformulated as follows:

\[
\begin{align*}
\min_{x \in \Omega(t)} & \quad f(x) \\
\text{s.t.} & \quad P_\xi(\phi(x)) < \log(10^{-4}) \\
& \quad E_\xi \left[ 1 - \exp \left( -e^{a_T \max_{t \in [0,T]} (x(t, \xi) - \theta_T)} \right) \right] < 10^{-4} \\
& \quad E_\xi \left[ 1 - \exp \left( -e^{b_T \max_{t \in [0,T]} (x(t, \xi) - \theta_T)} \right) \right] < 10^{-4}
\end{align*}
\]

The uncertainties on \( \xi \) can be dealt with Improved Monte Carlo methods (Importance Sampling, Subset Simulation...).

6-Optimization strategy

Different approaches:
- approaches based on FORM/SORM: double loop optimization[RIA, PMA] or single loop optimization (SORA, SAP \cite{3})
- meta-model with adaptive design of experiments + optimization \cite{4 5 6}
- Lagrangian formulation: \( \min_{x, \lambda} f(x) + \lambda \left( E_\xi[\text{constraints}(x, \xi)] \right) \)

7-Next step: add the Extreme constraint

Extreme constraint:
\[
P_\xi(\max_{t \in [0,T]} g_4(t, x, \xi; z) > 0) < 10^{-4}
\]

- \( g_4 \) is nonlinear and costly to evaluate \( \Rightarrow \) Extreme Value Theory unusable
- With truncated Karhunen-Loève expansion:
\[
\begin{align*}
P_\xi(x_1, \ldots, x_m) & \quad \max_{t \in [0,T]} \phi_k^R(t, x, \xi; z_1, \ldots, z_m) > 0 < 10^{-4}
\end{align*}
\]

References

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