Outlook

- Avalanche flow dynamic models depend on inputs that are poorly known.
- These models are employed for land-use planning as far as for the design of defense structures. It is thus important to assess the impact of the uncertainty of the input parameters on the outputs.
- Very few sensitivity analyses in the avalanche field have been developed.
- The outputs of these models are both functional and scalar.
- The functional outputs have a high number of zeros which corresponds to regions where there is no avalanche.
- The novelty: given a sample of runs, we calculate sensitivity indices of conditional random variables.

The avalanche model

The avalanche model represents the avalanche as a fluid in motion and is based on depth-averaged Saint Venant equations.

The inputs

- **Inputs Description Uncertainty interval**
  - $X_1 = \mu$: Static friction coefficient [0.1-0.65]
  - $X_2 = \xi$: Turbulent friction [m/s$^2$] [400-10000]
  - $X_3 = \text{Lend}$: Length of the release zone [m] [5-100]
  - $X_4 = \text{Hend}$: Snow depth in the release zone [m] [0.1-4]
  - $X_5 = \text{xstart}$: Release abscissa [m] [0.01-200]
  - $X_6 = \sigma$: Digital Elevation Model error [m] [0-0.15]

The outputs

- The functional velocity $v$ and the functional snow depth $h$ are discretized on $N_D$ points. The runout distance $x_{\text{runout}}$ is scalar. The output $Y$ is thus vectorial.

The Sobol’ indices

The random inputs $X_1, \ldots, X_6$ are supposed independent, $\mathbb{E}(|Y|^2) < \infty$.

- First-order Sobol’ indices $\forall i \in \{1, \ldots, d\}, \forall j \in \{1, \ldots, p\}$:
  $$S'_{ij} = \frac{\text{Var}(E(Y_j | Y_{-j} > 0, X_i))}{\text{Var}(Y_j | Y_{-j} > 0)}$$

- Second-order Sobol’ indices $\forall i \neq j \in \{1, \ldots, d\}, \forall j \in \{1, \ldots, p\}$:
  $$S'_{ij} = \frac{\text{Var}(E(Y_j | Y_{-j} > 0, X_i, X_j))}{\text{Var}(Y_j | Y_{-j} > 0)} - S'_{ij} - S'_{ji}$$

- Aggregated Sobol’ indices $\forall i \in \{1, \ldots, d\}$:
  $$S_{gi} = \frac{\sum_{j=1}^{p} \text{Var}(Y_j | Y_{-j} > 0)}{\sum_{j=1}^{p} \text{Var}(Y_j | Y_{-j} > 0)} S'_{ij}$$

The estimation methods: given data methods

- There are several methods to estimate the Sobol’ indices.
- The samples of $Y_j | Y_{-j} > 0$ are created using acceptance-rejection sampling.

Specific sampling design methods cannot be used.

- We use two methods: the one sample or given data method (NSD to code no specific design) and Effective algorithm for computing global sensitivity indices (EASI) method.
- NSD partitions the input space in bins and estimates the indices based on the bin conditional distribution.

EASI is a spectral method based on the Fast Fourier Transform to estimate $S'_{ij}$.

The application

A total of 200000 avalanche model simulations were run using a LHS design. To test the accuracy of the methods, non parametric 95% bootstrapping confidence intervals with the bias percentile method were computed using 30 samples.

The first order indices (NSD vs EASI)

The first and second order indices (NSD) of the velocity:

The aggregated order Sobol indices

Conclusions

- We developed a methodology to estimate scalar and aggregated Sobol indices and bootstrap intervals of acceptance-rejection samples.
- We obtained similar results using EASI and NSD.
- The NSD method is capable to compute first and second order indices.
- The friction parameter $\mu$ is the most important of the avalanche model but the other inputs are non negligible since they show variations along the path.

Perspectives

- To develop a given data methodology to estimate Shapley effects, which are more meaningful in the context of sensitivity analysis based on acceptance-rejection sampling, as the inputs are then confined to a non-rectangular domain.
- To study other non specific sampling methods to estimate the sensitivity indices of these particular model outputs (e.g., ANOVA kernels).
- To apply this methodology to other avalanche models and paths to generalize the results.

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References