Sensitivity analysis of an avalanche flow dynamic model

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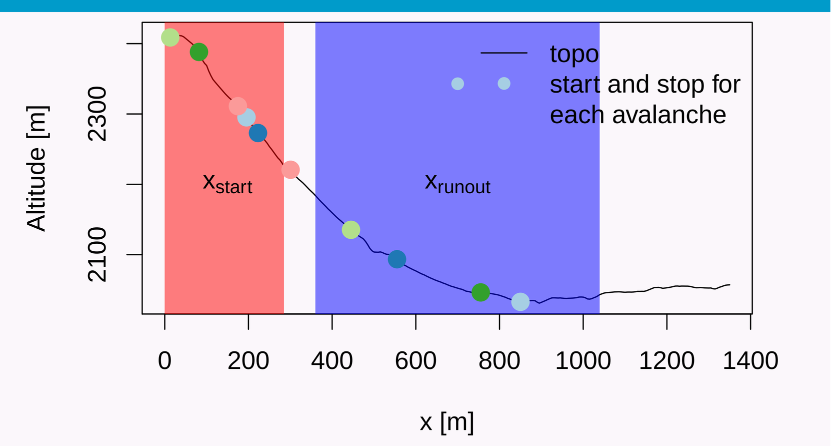
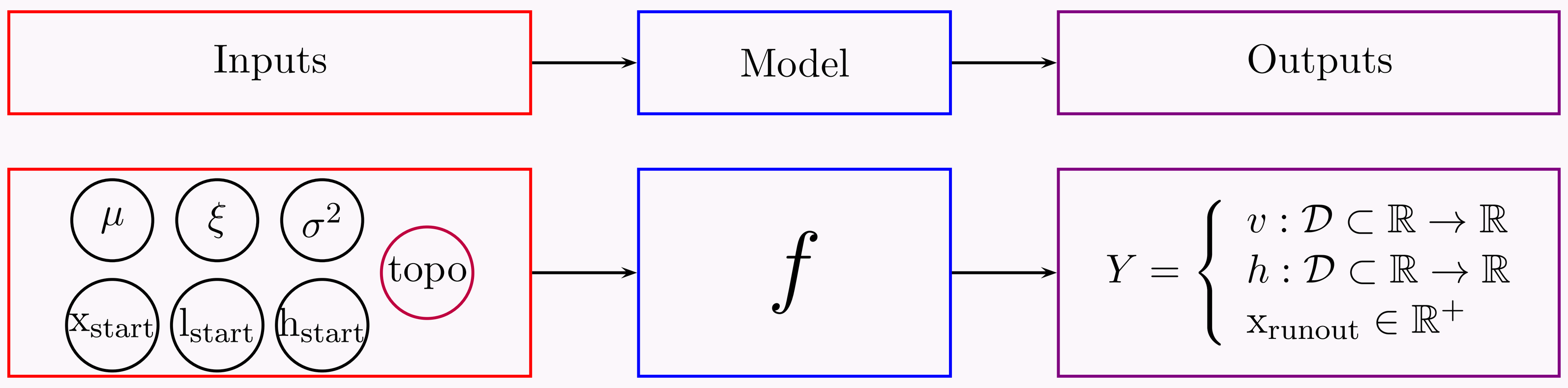
Outlook

- Avalanche flow dynamic models depend on inputs that are poorly known.
- These models are employed for land-use planning as far as for the design of defense structures. It is thus important to assess the impact of the uncertainty of the input parameters on the outputs.
- Very few sensitivity analyses in the avalanche field have been developed.
- The outputs of these models are both functional and scalar.
- The functional outputs have a high number of zeros which corresponds to regions where there is no avalanche.
- The novelty: given a sample of runs, we calculate sensitivity indices of conditional random variables.

The avalanche model

The avalanche model represents the avalanche as a fluid in motion and is based on depth-averaged Saint-Venant equations.





The inputs

Inputs	Description	Uncertainty interval
$X_1 = \mu$	Static friction coefficient	0.1-0.65
$X_2 = \xi$	Turbulent friction [m/s²]	400-10000
$X_3 = l_{start}$	Length of the release zone [m]	5-100
$X_4 = h_{start}$	Snow depth in the release zone [m]	0.1-4
$X_5 = x_{start}$	Release abscissa [m]	0.01-285
$X_6 = \sigma$	Digital Elevation Model error [m]	0-0.15

Uniform distributions are chosen to model the d=6 parameters denoted $X_1, \ldots, X_d$.

The outputs

The functional velocity v and the functional snow depth h are discretized on $N_{\mathcal{D}}$ points. The runout distance x_{runout} is scalar. The output $\mathbf{Y}$ is thus vectorial.

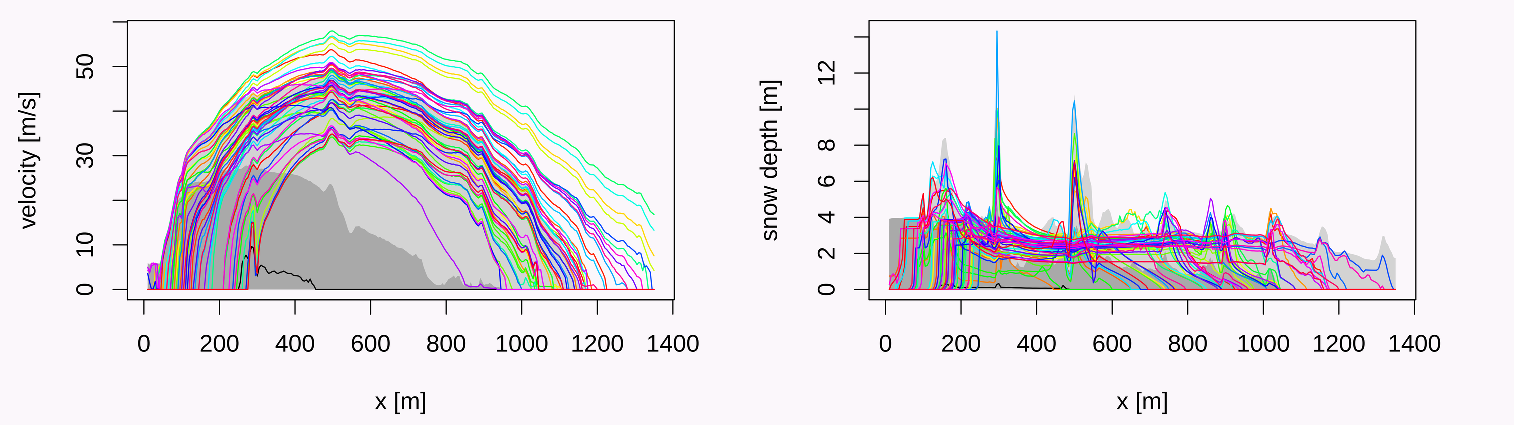


Figure: Functional high density region boxplots of velocity (left) and snow depth (right) of 1000 avalanche runs.

⚠ We are only interested in $Y_j | Y_j > 0$ $\forall j \in \{1, \ldots, p = 2 \times N_{\mathcal{D}} + 1\}$.
There is no avalanche if $Y_j = 0$.

The Sobol' indices

The random inputs $X_1, \ldots X_d$ are supposed independent, $\mathbb{E}(||\mathbf{Y}||^2) < \infty$.

First-order Sobol' indices $\forall i \in \{1, \ldots, d\}$, $\forall j \in \{1, \ldots, p\}$:

$$S_i^j = \frac{\mathrm{Var}(\mathbb{E}(Y_j | Y_j > 0, X_i))}{\mathrm{Var}(Y_j | Y_j > 0)} \quad (1)$$

Second-order Sobol' indices $\forall i_1 \neq i_2 \in \{1, \ldots, d\}$, $\forall j \in \{1, \ldots, p\}$:

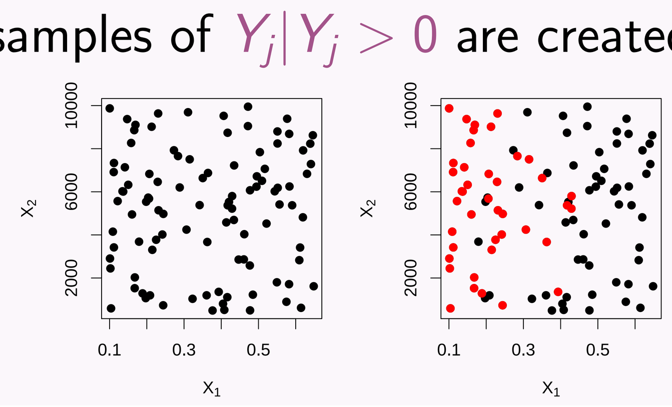
$$S_{i_1 i_2}^j = \frac{\mathrm{Var}(\mathbb{E}(Y_j | Y_j > 0, X_{i_1}, X_{i_2}))}{\mathrm{Var}(Y_j | Y_j > 0)} - S_{i_1}^j - S_{i_2}^j. \quad (2)$$

Aggregated Sobol' indices $\forall i \in \{1, \ldots, d\}$

$$GS_i = \frac{\sum_{j=1}^p \mathrm{Var}(Y_j | Y_j > 0) S_i^j}{\sum_{j=1}^p \mathrm{Var}(Y_j | Y_j > 0)} \quad (3)$$

The estimation methods: given data methods

- There are several methods to estimate the Sobol indices.
- The samples of $Y_j | Y_j > 0$ are created using acceptance-rejection sampling.



Original sample (left) and acceptance-rejection sample in red points (right) of a sample output.

- Specific sampling design methods cannot be used.
- We use two methods: the one sample or given data method (NSD to code no specific design) and Effective algorithm for computing global sensitivity indices (EASI) method.
  - NSD partitions the input space in bins and estimates the indices based on the bin conditional distribution.
  - EASI is a spectral method based on the Fast Fourier Transform to estimate S_i^j.

The application

A total of 200000 avalanche model simulations were run using a LHS design. To test the accuracy of the methods, non parametric 95% bootstrapping confidence intervals with the bias percentile method were computed using 30 samples.

The first order indices (NSD vs EASI)

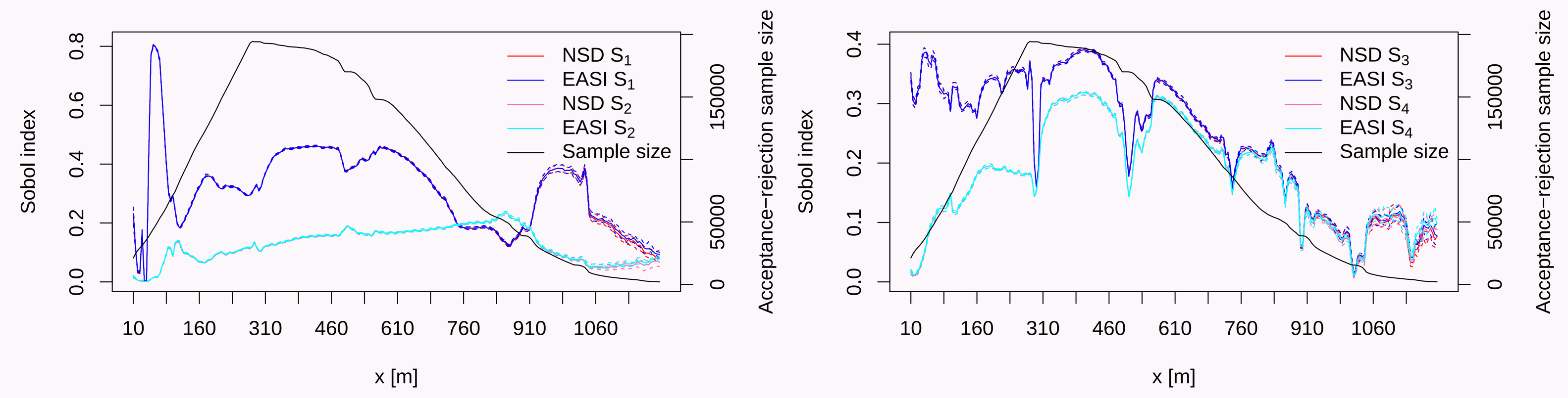
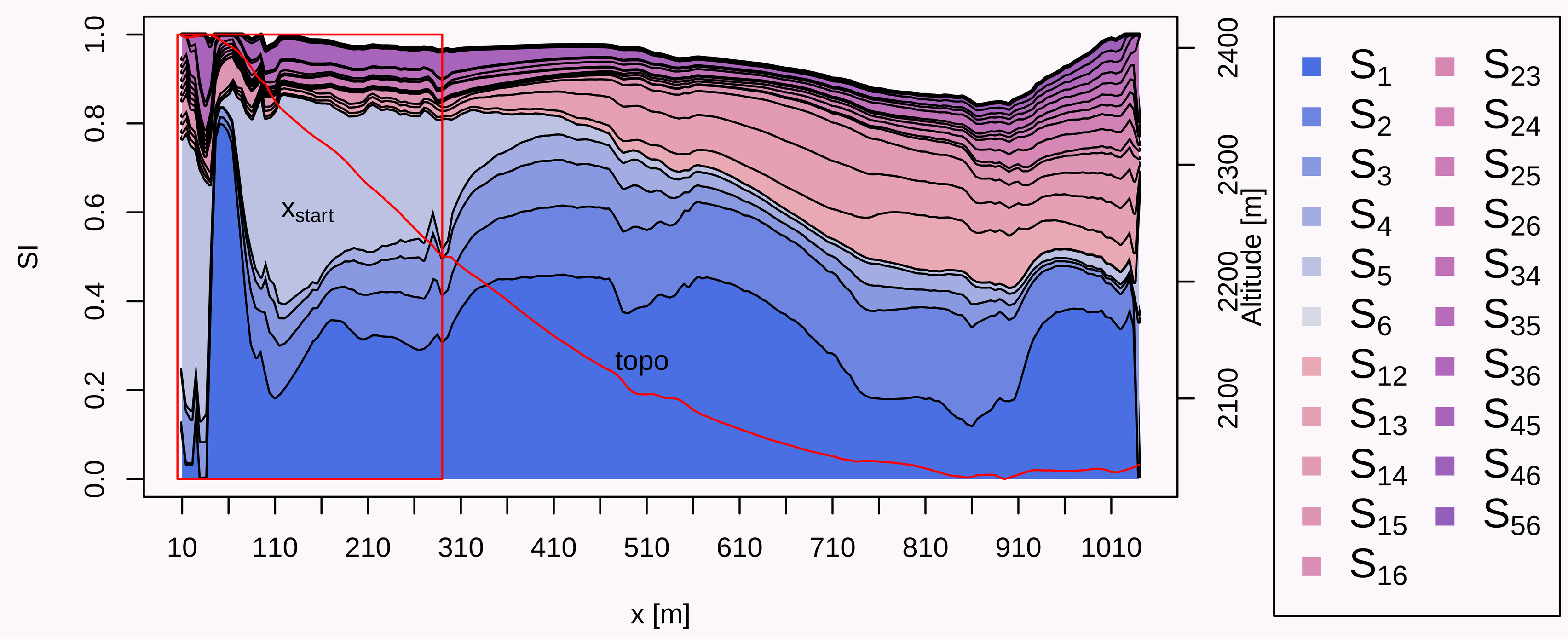
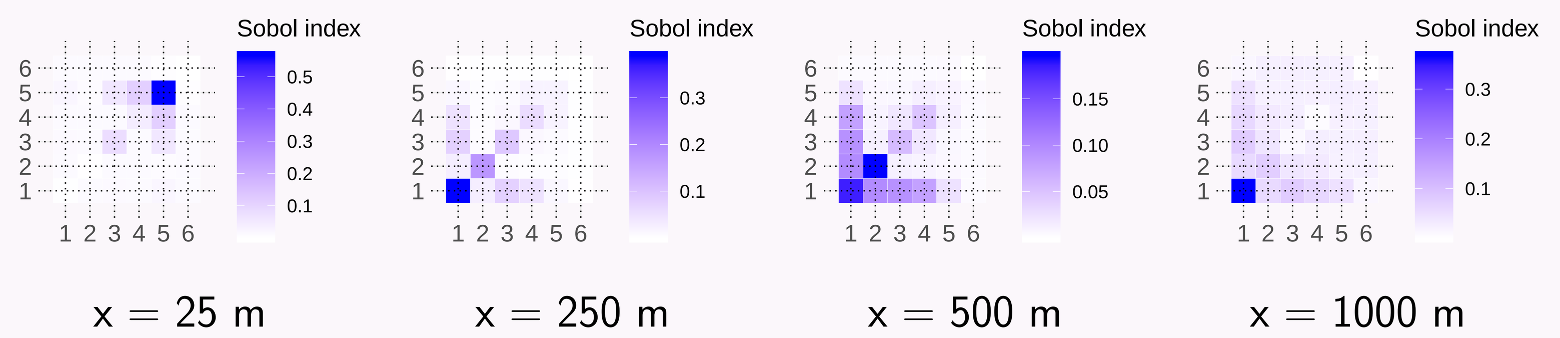


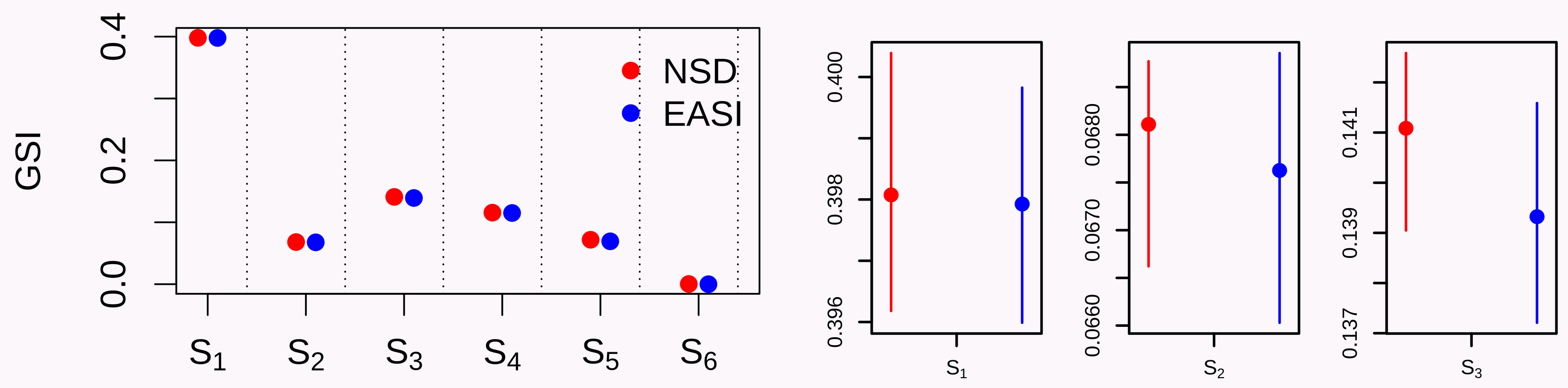
Figure: SI of the velocity curves (left) and SI of the snow depth curves (right).

The first and second order indices (NSD) of the velocity:





The aggregated order Sobol indices



Conclusions

- We developed a methodology to estimate scalar and aggregated Sobol indices and bootstrap intervals of acceptance-rejection samples.
- We obtained similar results using EASI and NSD.
- The NSD method is capable to compute first and second order indices.
- The friction parameter μ is the most important of the avalanche model but the other inputs are non negligible since they show variations along the path.

Perspectives

- To develop a given data methodology to estimate Shapley effects, which are more meaningful in the context of sensitivity analysis based on acceptance-rejection sampling, as the inputs are then confined to a non-rectangle domain.
- To study other no specific sampling methods to estimate the sensitivity indices of these particular model outputs (e.g., ANOVA kernels).
- To apply this methodology to other avalanche models and paths to generalize the results.

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