Mix Fleet Assignment of Hybrid and Electric Buses with Opportunity Charging

Francesco VITI, Marco RINALDI
Problem statement

eCoBus WP2: Charging/Backoffice Objectives

- Transit operational planning:
  - Network route design
  - Timetabling
  - Scheduling vehicles to trips
  - Driver assignment

- Fleet management
  - Well known in OR literature (Single/Multi Depot Vehicle Scheduling Problem)
  - Some existing approaches / algorithms to address problem including electrification
  - Under the assumption of **homogeneous fleet** and **deterministic travel times**
**Problem statement**

**CONVENTIONAL FLEET**

<table>
<thead>
<tr>
<th>Route/Stop</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
</tr>
</thead>
<tbody>
<tr>
<td>Gare Centrale - Steinl</td>
<td>9</td>
<td>7</td>
<td>10</td>
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</tbody>
</table>

*Executed, conventional*

**MIXED-FLEET**

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<tr>
<th>Route/Stop</th>
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*Executed, electric*

**FULL-ELECTRIC**

<table>
<thead>
<tr>
<th>Route/Stop</th>
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<th>2</th>
<th>3</th>
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*Executed, electric*

**DISADVANTAGES**
- High emissions;
- Policy / regulations preventing access to restricted areas.

**OPPORTUNITIES**
- Complement e-fleet while busy, targeting shorter trips;
- Substitute e-fleet during recharging operations.

**CHALLENGES**
- Large enough fleet, high investment;
- Trade-off between fleet size and delayed trips due to recharging.

**E-bus recharging**

**Delayed departure**
Contributions

Problem formulation

Single Terminal

Multi Terminal

Time-Lapse Decomposition

Test Results (planning)

Test Results (online//delays)
SDEVSP – Single Depot Electric Vehicle Scheduling Problem

<table>
<thead>
<tr>
<th>Var.</th>
<th>Domain</th>
<th>Explanation</th>
</tr>
</thead>
<tbody>
<tr>
<td>$y_{i,j}^t$</td>
<td>${0,1}$</td>
<td>1 if trip $j$ is initiated by e-bus $i$ at time $t$, 0 otherwise</td>
</tr>
<tr>
<td>$z_{h,j}^t$</td>
<td>${0,1}$</td>
<td>1 if trip $j$ is initiated by h-bus $i$ at time $t$, 0 otherwise</td>
</tr>
<tr>
<td>$x_{i,m}^t$</td>
<td>${0,1}$</td>
<td>1 if e-bus $i$ is being recharged at charging station $m$ at time $t$, 0 otherwise</td>
</tr>
<tr>
<td>$\varepsilon_i^t$</td>
<td>$\leq E_i \in \mathbb{R}^+$</td>
<td>Total energy in kWh that e-bus $i$ has at time $t$</td>
</tr>
<tr>
<td>$u_j$</td>
<td>$\mathbb{R}^+$</td>
<td>Total energy in kWh required to perform trip $j$</td>
</tr>
<tr>
<td>$d_j$</td>
<td>$\mathbb{R}^+$</td>
<td>Preferred departure time for trip $j$</td>
</tr>
<tr>
<td>$t_j$</td>
<td>$\mathbb{R}^+$</td>
<td>Duration of trip $j$ in time steps</td>
</tr>
<tr>
<td>$s_i^t$</td>
<td>$\mathbb{R}^+$</td>
<td>Slack variable, necessary to ensure that constraint (11) does not violate the domain of $\varepsilon_i^t$</td>
</tr>
<tr>
<td>$E$</td>
<td>$\mathbb{R}^+$</td>
<td>Total battery capacity in kWh for all electric buses</td>
</tr>
<tr>
<td>$A_i^t$</td>
<td>${0,1}$</td>
<td>1 if e-bus $i$ is not available to perform any trip of the current window problem at time $t$, 0 otherwise</td>
</tr>
<tr>
<td>$H_i^t$</td>
<td>${0,1}$</td>
<td>1 if h-bus $i$ is not available to perform any trip of the current window problem at time $t$, 0 otherwise</td>
</tr>
</tbody>
</table>
Methodology (S)

MILP Formulation: objective function

\[
\min \sum_{t} \sum_{i} \sum_{j} \gamma_{i,j}^{t} \cdot \left( c + r \cdot (t - d_{j}) \right) + \\
\sum_{t} \sum_{h} \sum_{j} \zeta_{h,j}^{t} \cdot \left( \hat{c} + r \cdot (t - d_{j}) \right) + \\
\sum_{m} \sum_{i} \sum_{t} q_{i}^{t} \cdot x_{i,m}^{t}
\]

- Blue: Cost of executing trip j with an e-bus
- Orange: Cost of executing trip j with an h-bus
- Green: Total cost of recharging
- Red: Late execution penalty term
Methodology (S)

MILP Formulation: constraints

\[ \sum_{j} y_{i,j}^{t} + \sum_{m} x_{i,m}^{t} \leq 1 \ \forall i, t \]

\[ y_{i,j}^{t} + \frac{1}{t_{j} - 1} \sum_{\bar{t} = t+1}^{t+t_{j}-1} \left( \sum_{\bar{j}} \bar{y}_{i,\bar{j}}^{\bar{t}} + \sum_{m} \bar{x}_{i,m}^{\bar{t}} \right) \leq 1 \ \forall i, \forall j : t_{j} > 1, \forall t \]

\[ \sum_{j} z_{h,j}^{t} \leq 1 \ \forall h, t \]

\[ z_{h,j}^{t} + \frac{1}{t_{j} - 1} \sum_{\bar{t} = t+1}^{t+t_{j}-1} \sum_{\bar{j}} z_{h,\bar{j}}^{\bar{t}} \leq 1 \ \forall h, \forall j : t_{j} > 1, \forall t \]

\[ \sum_{t} \left( \sum_{i} y_{i,j}^{t} + \sum_{h} z_{h,j}^{t} \right) = 1 \ \forall j \]

\[ \sum_{t < d_{j}} \left( \sum_{i} y_{i,j}^{t} + \sum_{h} z_{h,j}^{t} \right) = 0 \ \forall j \]

Service constraints
- Schedule adherence
- Trip execution
- Feasibility
Methodology (S)

MILP Formulation: constraints

Energy constraints
- Electricity consumption
- Discharging/recharging dynamics

\[
\begin{align*}
    y_{i,j}^t - \frac{\varepsilon_i^t}{u_j + \mu E} &\leq 0 \quad \forall i, j, \forall t \\
    \sum_{i} x_{i,m}^t &\leq 1 \quad \forall m, t \\
    \varepsilon_i^0 &= \bar{\varepsilon}_i \quad \forall i \\
    E \cdot \sum_{m} x_{i,m}^t - \sum_{j} y_{i,j}^t \cdot u_j + \varepsilon_i^t - s_i^t &= \varepsilon_i^{t+1} \quad \forall i, t \\
    \sum_{m} x_{im}^t - \frac{s_i^t}{E} &\geq 0 \quad \forall i, t \\
    \frac{1}{E} \cdot s_i^t - \frac{1}{E} \varepsilon_i^t &\leq 0 \quad \forall i, t
\end{align*}
\]
Experimental results

Test case (1): Gare Centrale
Experimental results
Single Terminal

Gare Centrale

Operational cost [EUR]

Fleet composition [% e-bus]

Total # of recharges / day

ObjFun[15 EUR]
ObjFun[0.15 EUR]
# Recharges [15 EUR]
# Recharges [0.15 EUR]
Experimental results

Test case (2): Bouillon P+R
Experimental results

Single Terminal

Bouillon P+R

---

[Graph showing operational cost and total number of recharges vs. fleet composition]
Methodology (M)
MILP Formulation: objective function

\[
\min \sum_{t} \sum_{i} \sum_{j} c \cdot (1 + r \cdot (t - d_j)) \cdot y_{i,j} + \\
\sum_{t} \sum_{h} \sum_{j} \hat{c} \cdot (1 + r \cdot (t - d_j)) \cdot z_{h,j} + \\
\sum_{t} \sum_{i} \sum_{b_1} \sum_{b_2} \hat{c} \cdot \omega_{i,b_1,b_2} + \\
\sum_{i} \sum_{t} \sum_{b} \sum_{m} q_i \cdot x_{i,b,m}
\]

- Cost of executing trip j with an e-bus
- Cost of executing trip j with an h-bus
- Cost of executing deadheading trip w
- Total cost of recharging
- Late execution penalty term
Methodology (M)
MILP Formulation: constraints

\[
\sum_{j} y_{i,j}^{t} + \sum_{b_{1},b_{2} \in B} \sum_{t} \omega_{i,b_{1},b_{2}}^{t} + \sum_{m} x_{i,m}^{t} \leq 1 - A_{i}^{t} \quad \forall i, t
\]

\[
y_{i,j}^{t} = \frac{1}{t_{j} - 1} \sum_{t = t+1}^{t+t_{j}-1} \left( \sum_{j} y_{i,j}^{t+1} \right) \leq 1 \quad \forall i, \forall j : t_{j} \quad 1, \forall t : d_{j} \leq t \leq d_{j} + \theta
\]

\[
z_{h,j}^{t} + \frac{1}{t_{j} - 1} \sum_{t = t+1}^{t+t_{j}-1} \sum_{j} z_{h,j}^{t+1} \leq 1 \quad \forall h, \forall j : t_{j} > 1, \forall t
\]

\[
\sum_{j} z_{h,j}^{t} \leq 1 - H_{h}^{t} \quad \forall h, t
\]

\[
\omega_{i,b_{1},b_{2}}^{t} = \frac{1}{t_{b} - 1} \sum_{t = t+1}^{t+t_{b}-1} \left( \sum_{j} y_{i,j}^{t+1} \right) \leq 1 \quad \forall i, \forall b_{1} \in B, \forall b_{2} \in B, \forall t : d_{b}
\]

\[
\sum_{t} \left( \sum_{i} y_{i,j}^{t} + \sum_{h} z_{h,j}^{t} \right) = 1 \quad \forall j
\]

\[
\sum_{t : d_{j} \cup t > d_{j} + \theta} \left( \sum_{i} y_{i,j}^{t} + \sum_{h} z_{h,j}^{t} \right) = 0 \quad \forall j
\]

Service constraints
- Schedule adherence
- Trip execution
- Deadhead trip execution
- Feasibility
Methodology (M)

MILP Formulation: constraints

Service constraints

- Bus location updating

\[
\sum_{m} x_{i,b,m}^t - g_{i,b}^t \leq 0 \ \forall t,i, \forall b \in B
\]

\[
\sum_{j: \alpha_j = b_1} y_{i,j}^t + \sum_{b_2} \omega_{i,b_1,b_2}^t - g_{i,b_1}^t \leq 0 \ \forall i, b_1, t
\]

\[
\sum_{j: \beta_j = b_2} y_{i,j}^t + \sum_{b_1} \omega_{i,b_1,b_2}^{t+1} - g_{i,b_2}^{t+1} \leq 0 \ \forall i, b_2, t
\]

\[
\sum_{j: \beta_j = b_2} y_{i,j}^t + \sum_{b_1} \omega_{i,b_1,b_2}^t - (g_{i,b_2}^{t+1} - g_{i,b_2}^t) \geq 0 \ \forall i, b_2, t
\]

\[
\sum_{b} g_{i,b}^t = 1 \ \forall i, t
\]

\[
g_{i,b}^0 = \begin{cases} 
1 & \text{if } b = G_i \\
0 & \text{otherwise}
\end{cases} \ \forall i, b
\]

\[
\sum_{j: \alpha_j = b} z_{h,j}^t - p_{h,b}^t \leq 0 \ \forall h,b,t
\]

\[
\sum_{j: \beta_j = b} z_{h,j}^t - (p_{h,b}^{t+1} - p_{h,b}^t) \geq 0 \ \forall h,b,t
\]

\[
\sum_{b} p_{h,b}^t = 1 \ \forall h, t
\]

\[
p_{h,b}^0 = \begin{cases} 
1 & \text{if } b = P_h \\
0 & \text{otherwise}
\end{cases} \ \forall h, b
\]
Methodology (M)

MILP Formulation: constraints

\[\begin{align*}
    y_{i,j}^t - \frac{\varepsilon_i^t}{u_j} + \min_{B_j \notin B, b_2 \in B} (\hat{u}_{j,b_2}) + \mu E &\leq 0 \quad \forall i, j, \forall t : d_j \leq t \leq d_j + \theta \\
    w_{i,b_1,b_2}^t - \frac{\varepsilon_i^t}{\hat{u}_{b_1,b_2}^t} &\leq 0 \quad \forall i, j, t, b_1, b_2
\end{align*}\]

Energy constraints
- Electricity consumption
- Discharging/recharging dynamics

\[\sum_i x_{i,b,m}^t \leq 1 \quad \forall m, t, b \in B\]

\[\varepsilon_i^0 = \bar{\varepsilon}_i \quad \forall i\]

\[E \cdot \sum_{b \in B} \sum_m x_{i,b,m}^t - \sum_j y_{i,j}^t \cdot u_j - \sum_{b_1, b_2 \in B} \sum_t \omega_{i,b_1,b_2}^t \cdot \hat{u}_{b_1,b_2}^t + \varepsilon_i^t - s_i^t = \varepsilon_i^{t+1} \quad \forall i, t\]

\[\sum_{b \in B} \sum_m x_{i,b,m}^t - \frac{s_i^t}{E} \geq 0 \quad \forall i, t\]

\[\frac{1}{E} s_i^t - \frac{1}{E} \varepsilon_i^t \leq 0 \quad \forall i, t\]
Experimental results

Test case (4): multi-terminal
Experimental results
Multi Terminal: Fleet composition

![Graph showing the relationship between the percentage of electric buses (% E-buses) and total operational cost (EUR) vs. total amount of charging operations. The graph displays two lines: one indicating a decrease in total operational cost as the percentage of electric buses increases, and the other showing an increase in total amount of charging operations with a decrease in the percentage of electric buses.](image-url)
Experimental results
Multi Terminal: Fleet composition (cost factors)
Relationship to TCO
Linear cost regression results (50% resale)

Mixed-fleet replacement cost projection, assuming a hybrid-bus resale price of 50%

Latest break-even point: ~6y3mo
Relationship to TCO
Linear cost regression results (30% resale)

Mixed-fleet replacement cost projection, assuming a hybrid-bus resale price of 30%

Latest break-even point: ~8y
Thank you for your attention!

{francesco.viti;marco.rinaldi}@uni.lu
Experimental setup

Online//Delay validation
Experimental results

Test results (3)

<table>
<thead>
<tr>
<th>Demand [pax/h/line]</th>
<th>300 pax/h/line</th>
<th>500 pax/h/line</th>
<th>700 pax/h/line</th>
<th>900 pax/h/line</th>
</tr>
</thead>
<tbody>
<tr>
<td>% delayed trips</td>
<td>85.00 base</td>
<td>82.50 control</td>
<td>88.33 base</td>
<td>86.77 control</td>
</tr>
<tr>
<td></td>
<td>88.33 base</td>
<td>83.33 control</td>
<td>88.33 base</td>
<td>86.67 control</td>
</tr>
<tr>
<td># Rescheduled EBs</td>
<td>62 base</td>
<td>67 control</td>
<td>64 base</td>
<td>64 control</td>
</tr>
<tr>
<td></td>
<td>66 base</td>
<td>64 control</td>
<td>66 base</td>
<td>64 control</td>
</tr>
<tr>
<td># Rescheduled EB to HB</td>
<td>22 base</td>
<td>21 control</td>
<td>26 base</td>
<td>25 control</td>
</tr>
<tr>
<td></td>
<td>29 base</td>
<td>26 control</td>
<td>29 base</td>
<td>26 control</td>
</tr>
<tr>
<td>Avg. Trip Delay [s]</td>
<td>215.20 base</td>
<td>216.36 control</td>
<td>348.02 base</td>
<td>368.11 control</td>
</tr>
<tr>
<td></td>
<td>526.90 base</td>
<td>517.61 control</td>
<td>714.37 base</td>
<td>678.31 control</td>
</tr>
<tr>
<td>Avg. Bunching level [%]</td>
<td>0.93 base</td>
<td>0.91 control</td>
<td>0.91 base</td>
<td>0.91 control</td>
</tr>
<tr>
<td></td>
<td>0.91 base</td>
<td>0.89 control</td>
<td>0.91 base</td>
<td>0.91 control</td>
</tr>
<tr>
<td>Avg. Arrival CV [%]</td>
<td>0.39 base</td>
<td>0.39 control</td>
<td>0.49 base</td>
<td>0.49 control</td>
</tr>
<tr>
<td></td>
<td>0.54 base</td>
<td>0.54 control</td>
<td>0.54 base</td>
<td>0.54 control</td>
</tr>
</tbody>
</table>

\[ c = 0.459 \text{ EUR} / \text{km} \]
\[ \hat{c} = 0.621 \text{ EUR} / \text{km} \]
\[ q = 0.15 \text{ EUR} / \text{kWh} \]
References

- Rinaldi et al. (2018) – *Optimal dispatching of electric and hybrid buses subject to scheduling and charging constraints*, in Proceedings of the 21st IEEE International Conference on Intelligent Transportation Systems


- Picarelli et al. (2019) – Model and solution methods for the Mixed-Fleet Multi-Terminal Bus Scheduling Problem, in Transportation Research Procedia (22nd Euro Working Group on Transportation Meeting) [to appear]